

Time Value of Money

Definitions and formulas:

Single cash flow:

$$\text{Future value} = PV * (1 + i)^n$$

$$\text{Present value} = \frac{FV_n}{(1 + i)^n}$$

Multiple cash flows:

- **Ordinary annuity:** is a periodic equal cash flow **A** over **n** periods, occurring at the end of each period.

$$\text{Future value} = A * \left[\frac{(1 + i)^n - 1}{i} \right]$$

$$\text{Present value} = A * \left[\frac{1 - \frac{1}{(1 + i)^n}}{i} \right]$$

- **Growth Annuity:** is an annuity with a percentage of growth at a constant rate “**g**” over **n** periods, computed on the basis of the cash flow **A** at the end of the first period.

$$\text{Present value} = A * \left[\frac{1 - \left(\frac{1 + g}{1 + i} \right)^n}{i - g} \right]$$

- **Perpetuity:** is an ordinary annuity with an infinite maturity.

$$\text{Present value} = \frac{A}{i}$$

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- **Growth Perpetuity:** is a perpetuity with a percentage of growth at a constant rate “g” per period, computed on the basis of the cash flow A at the end of the first period.

$$\text{Present value} = A * \frac{1}{i - g}$$

Nominal (APR) and effective annual rates: i_e & i_n :

$$i_e = \left(1 + \frac{i_n}{m}\right)^m - 1$$

where m is the number of periods within one year.

Note: $i_n = APR$ (annual percentage rate).

Basic exercises (I-II):

1. Happy Harry has just bought a scratch lottery ticket and won €10,000. He wants to finance the future study of his newly born daughter and invests this money in a fund with a maturity of 18 years offering a promising yearly return of 6%. What is the amount available on the 18th birthday of his daughter?
2. Rudy will retire in 20 years. This year he wants to fund an amount of €15,000 to become available in 20 years. How much does he have to deposit into a pension plan earning 7% annually?
3. The National Savings Fund promises a monthly 0.75% return if you deposit €100 per month for 15 consecutive years. What amount will be accumulated after those 15 years?
4. Willy has just bought a house. She estimates that the roof will have to be renewed at a cost of €25,000 after 20 years. To cover these costs, she intends to save an equal amount of money at the end of each year, earning 6% annual interest rate. How much is such a yearly annuity?
5. Pete considers buying a house. Currently, he rents a place for €1,000 a month. The current monthly interest rate on mortgages is 0.5%. His planning period is 20 years. If he doesn't want to increase his housing costs, what amount of mortgage is available for his purchase? (Neglect any tax effects here).

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6. Liphips Ltd has just paid a dividend per share of €1.20. Shares are valued only on the basis of expected dividends. An annual sustainable growth of dividends of 4% is assumed. The appropriate discount rate (i) is 10% per year. The planning horizon is limited to 20 years. Compute the share value.
7. Compute the share value of a company paying a dividend of €3.60 per year over infinite maturity, with expected zero growth. The discount rate (i) is assumed to be 12% yearly.
8. Centuries ago, rich families in the province of Friesland established a fund to further welfare and education. From this fund, only the interest revenue was allowed to be spent, in order to keep the principal unattached. Assume the fund has amounted to €12 million and market interest rate (i) is 6% annually. What would be the perpetuity (or present value of the fund) endowed to the society?
9. Calculate the value of a constant cash flow of €500 a year with a growth of 4%, measured over an infinite period at a discount rate (i) of 10%.
10. Compute the effective yearly rate if the monthly rate is 1%.
11. Compute the quarterly interest rate concerning an effective annual rate of 12% and a nominal annual rate of 12%.

Advanced exercise (12):

12. A company buys a piece of equipment for €2 million on January 1. The expected useful life is 6 years and the salvage value is estimated zero. The company intends to replace the equipment identically. The average expected price increase is 8% yearly. For this purpose, the company creates a special fund with annual equal payments at the end of each year during the lifetime. Cost of capital and earnings of the fund (i) is 10% per year. Compute the annual payment into the fund.

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